Goodness-of-Fit Tests for Logistic Regression with Complex Survey Data

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Overview

- Background on weighting adjustment for nonresponse using response propensity scores
- Methodology on goodness-of-fit test for response propensity model
- Simulation study to investigate model misspecification
- Conclusion

Background

- Weighting adjustment

$$\hat{\theta}_A = \sum_{i \in R} \frac{w_i}{a_i} Y_i$$

Calculating adjustment factor:

$$a_{i(c)} = \sum_{i \in R_c} w_{i(c)} \Big/ \sum_{i \in S_c} w_{i(c)}$$

$$a_i = \hat{\pi}_i, \quad \log\left[\pi_i / \left(1 - \pi_i\right)\right] = X_i \beta$$

Background

- Weighting adjustment for nonresponse using response propensity model
  - individual estimate of propensity score as adjustment factor
  - weighting cells based on estimated propensity scores
- Both techniques depend on estimates of propensity scores
- Estimated propensity scores depend on goodness-of-fit of model

Background

- Goodness-of-fit test:
  - Hosmer-Lemeshow (H-L) test for simple random samples, available in SAS (unweighted)
  - for complex samples, available in SUDAAN and STATA (design-based, different in rejection regions)
- Effect of model misspecification
  - goodness-of-fit test
  - distribution of propensity scores
  - weighting cells

Methodology

- Goodness-of-fit test

  H_0: model fit is good
  H_1: model fit is not good

- Test for simple random sample

$$HL = \sum_{c=1}^{G} \frac{\left(O_c - N_c \bar{\pi}_c\right)^2}{N_c \bar{\pi}_c \left(1 - \bar{\pi}_c\right)} \sim \chi^2_{(G-2)}$$

Methodology

- Test for complex sample

$$\hat{\theta} = (O_1 - E_1, \cdots, O_G - E_G)$$

$$O_c - E_c = \sum_{i \in S_c} w_i (R_i - \hat{\pi}_i)$$

(SUDAAN)

$$O_c - E_c = \sum_{i \in S_c} w_i (R_i - \hat{\pi}_i) \Big/ \sum_{i \in S_c} w_i$$

(STATA)

$$Q = \hat{\theta}' \left[\text{var}(\hat{\theta}) \right]^{-1} \hat{\theta}$$

Methodology

- Rejection regions
  - SUDAAN Chi-square $Q \sim \chi^2_{(G-2)}$
  - SUDAAN Satterthwaite Adjusted *F*

$$\frac{G}{\bar{\lambda}(1+a^2)^2} Q^* \sim F^{G/(1+a^2)}_{K-G+2}, \quad \text{where } K = \#\text{PSUs} - \#\text{strata}$$

  - SUDAAN Wald *F* $\frac{Q}{G} \sim F^{G-1}_{K}$
  - STATA Wald *F* $\frac{K-G+2}{GK} Q \sim F^{G}_{K-G+2}$

Simulation Study

- Goals
  - to understand available GOF tests for complex survey data
  - to understand effect of model misspecification on GOF test
  - to understand effect of model misspecification on weighting adjustment

Simulation Study

- Data generation
  - 5 covariates (categorical): *X, Y, Z, U, V*
  - *X, Y, U, V* are independent; *Z* is highly correlated with *Y* but not with others

Covariate	Value	Probability
X	1, 2, 3	.5, .3, .2
Y	1, 2, 3, 4	.25, .25, .25, .25
Z	1, 2, 3, 4, 5	*Y* + Bernoulli(.5)
U	1, 2	.6, .4
V	1, 2, 3	.65, .25, .1

Simulation Study

- Data generation (continued)
  - Regression coefficients

$$\log\left[\pi/(1-\pi)\right] = \beta_0 + \beta_1 X + \beta_2 Y + \beta_3 Z + \beta_4 U + \beta_5 V$$

$$\beta_0 = -6,\ \beta_1 = 2.5,\ \beta_2 = 0.05,\ \beta_3 = 0,\ \beta_4 = 1.5,\ \beta_5 = -1.5$$

  - Response propensity and respondent indicator

$$\pi = \text{response propensity} = \frac{e^{X\beta}}{1+e^{X\beta}}$$

$$R = \text{respondent indicator} \sim Bernoulli(\pi)$$

Simulation Study

- Data generation (continued)
  - Sampling strata (*S*): Corr(*S*,*X*) is high
  - Sampling weight (*W*) depends on *X*

X	*W*
1	66
2	79
3	406

  - Sample size = 50,000; replicates = 1,000

Simulation Study

- Model misspecification
  - true model: $$\log\left(\frac{\pi}{1-\pi}\right) = \beta_0 + \beta_1 X + \beta_2 Y + \beta_4 U + \beta_5 V$$
  - overspecified model: $$\log\left(\frac{\pi}{1-\pi}\right) = \beta_0 + \beta_1 X + \beta_2 Y + \beta_3 Z + \beta_4 U + \beta_5 V$$
  - underspecified model: $$\log\left(\frac{\pi}{1-\pi}\right) = \beta_0 + \beta_2 Y + \beta_4 U + \beta_5 V$$

Simulation Study

- Evaluation:
  - estimates of regression coefficients
  - goodness-of-fit test's *p*-values
  - response propensity score distribution and weighting cells construction
- Software: SAS, SUDAAN, STATA

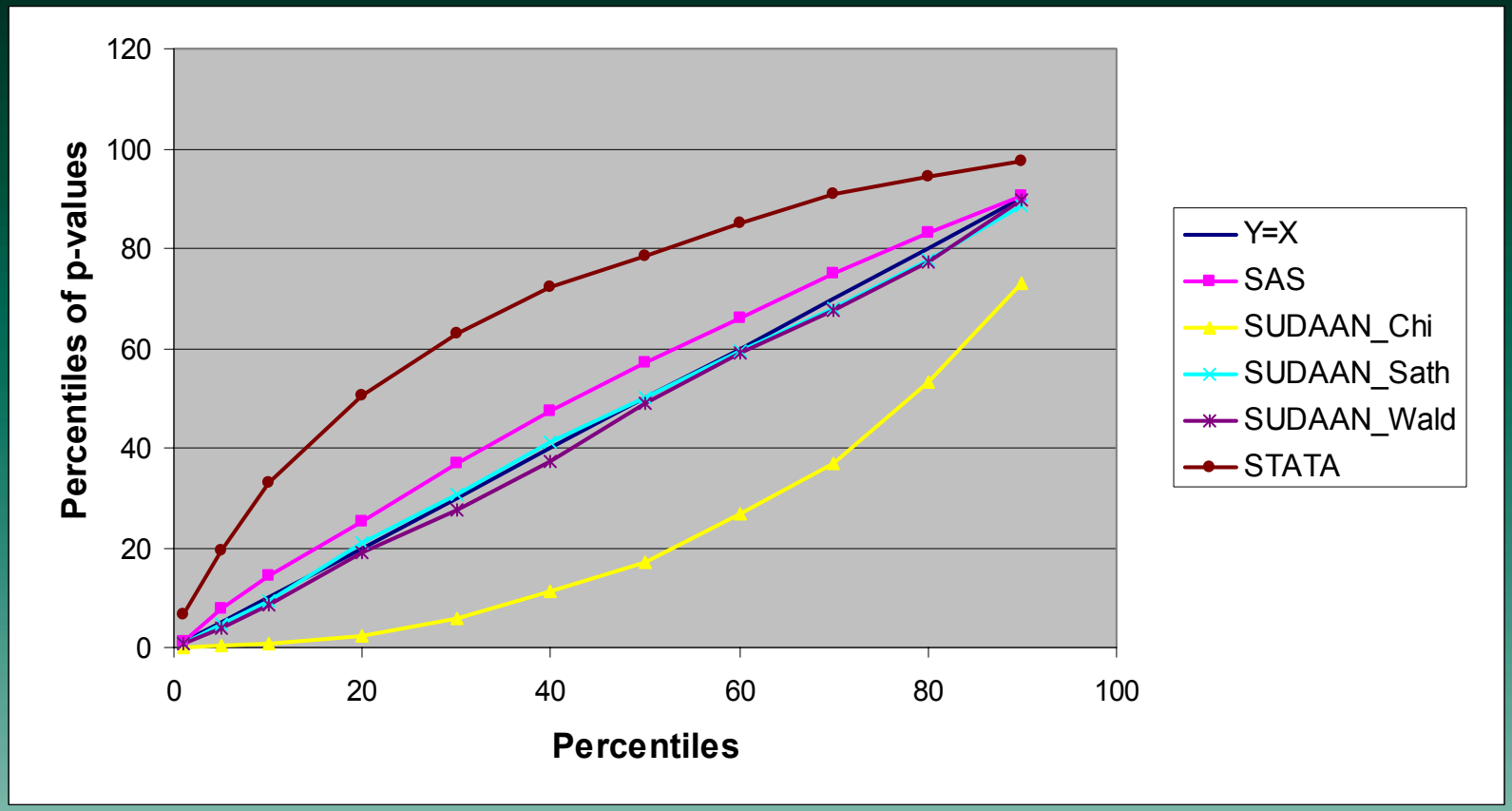
Simulation Study

Means of Regression Coefficients

BETA	True coefficient	True model			Overspecified model			Underspecified model		
		SAS	SUDAAN	STATA	SAS	SUDAAN	STATA	SAS	SUDAAN	STATA
Beta_0	-3.5	-3.50163	-3.50112	-3.50101	-3.50178	-3.50137	-3.50137	-2.5162	-1.90706	-1.90706
X = 2	1	0.9995	0.99938	0.99938	0.99964	0.99963	0.99963			
X = 3	2	2.00074	2.00052	2.00052	2.00097	2.00095	2.00095			
Y = 2	0.5	0.50164	0.50003	0.50003	0.50749	0.51145	0.51145	0.44943	0.44756	0.44756
Y = 3	1	1.00052	0.99913	0.99913	1.00899	1.01524	1.01524	0.90045	0.8982	0.8982
Y = 4	1.5	1.49982	1.49933	1.49933	1.50556	1.51638	1.51638	1.35128	1.34807	1.34807
Z = 2	0				-0.00591	-0.01161	-0.01161			
Z = 3	0				-0.00877	-0.01639	-0.01639			
Z = 4	0				-0.00563	-0.01697	-0.01697			
Z = 5	0				-0.00963	-0.02295	-0.02295			
U = 2	2.5	2.50145	2.50128	2.50128	2.50173	2.50185	2.50185	2.23122	2.22006	2.22006
V = 2	-1.5	-1.50051	-1.49863	-1.49863	-1.50068	-1.49895	-1.49895	-1.34454	-1.33848	-1.33848
V = 3	-3	-3.00051	-2.9975	-2.99746	-3.00082	-2.99822	-2.99822	-2.70838	-2.70305	-2.70305

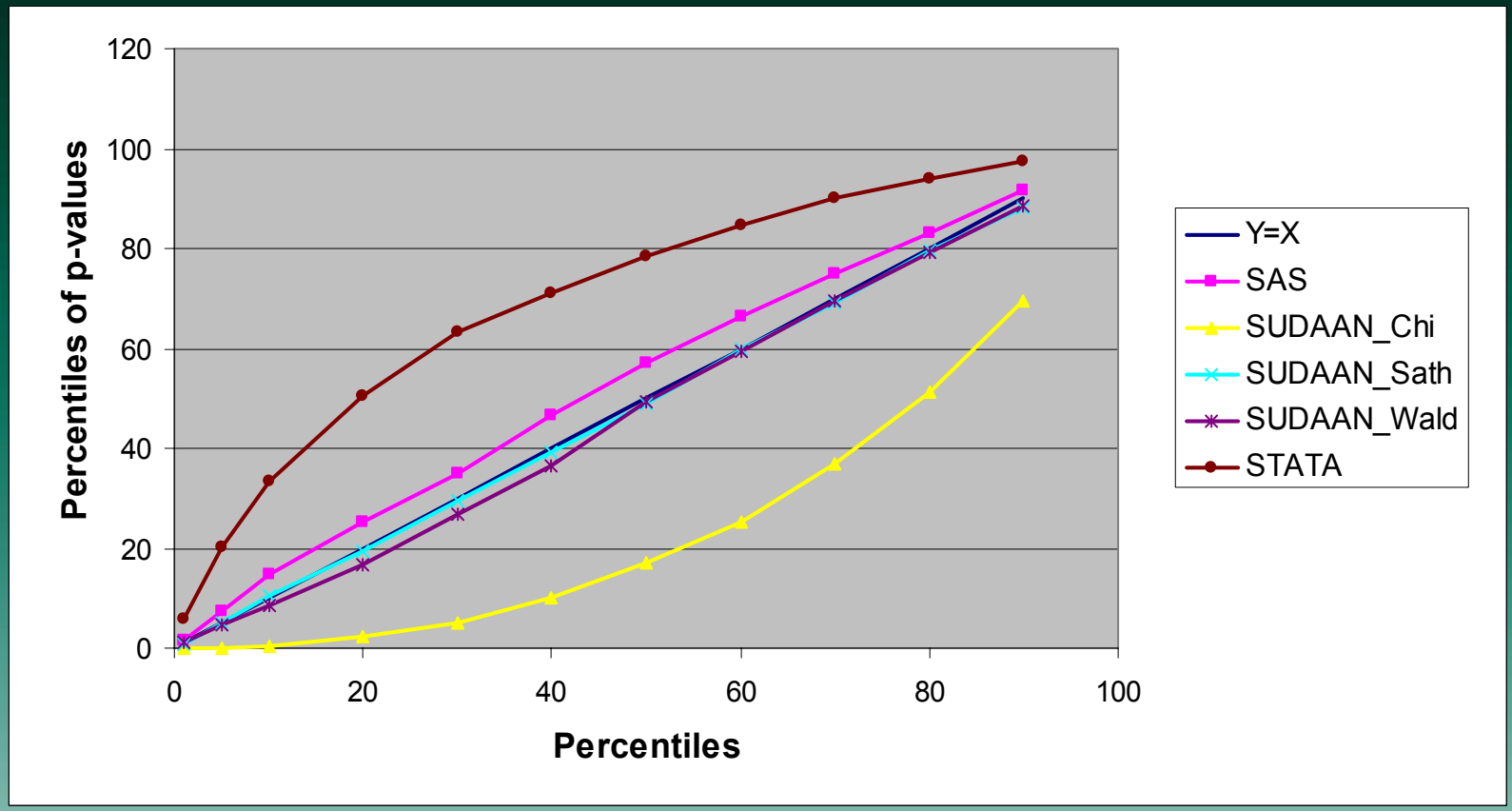
Simulation Study

Percentiles for *p*-values of True Model



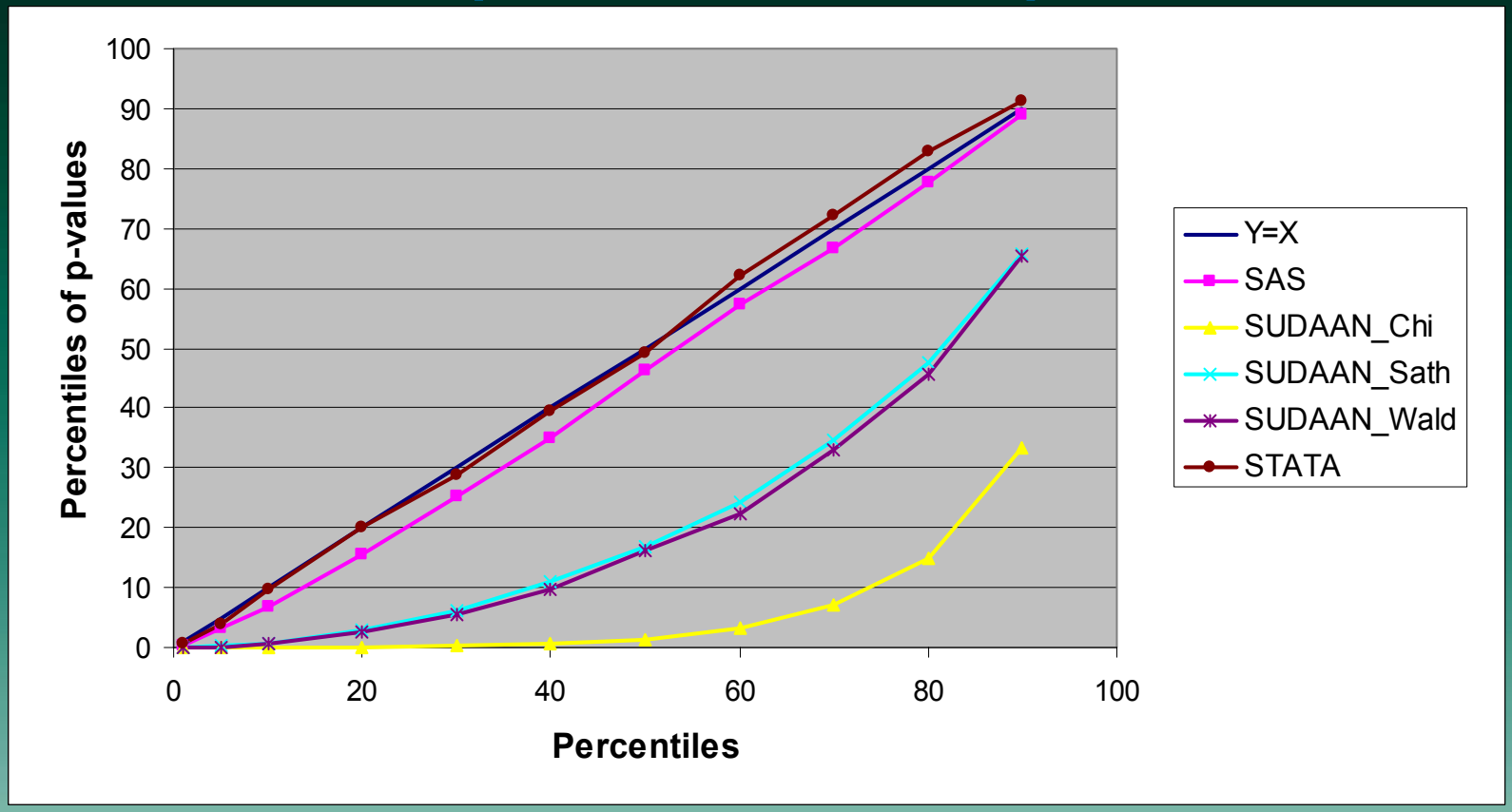
Simulation Study

Percentiles for *p*-values of Overspecified Model



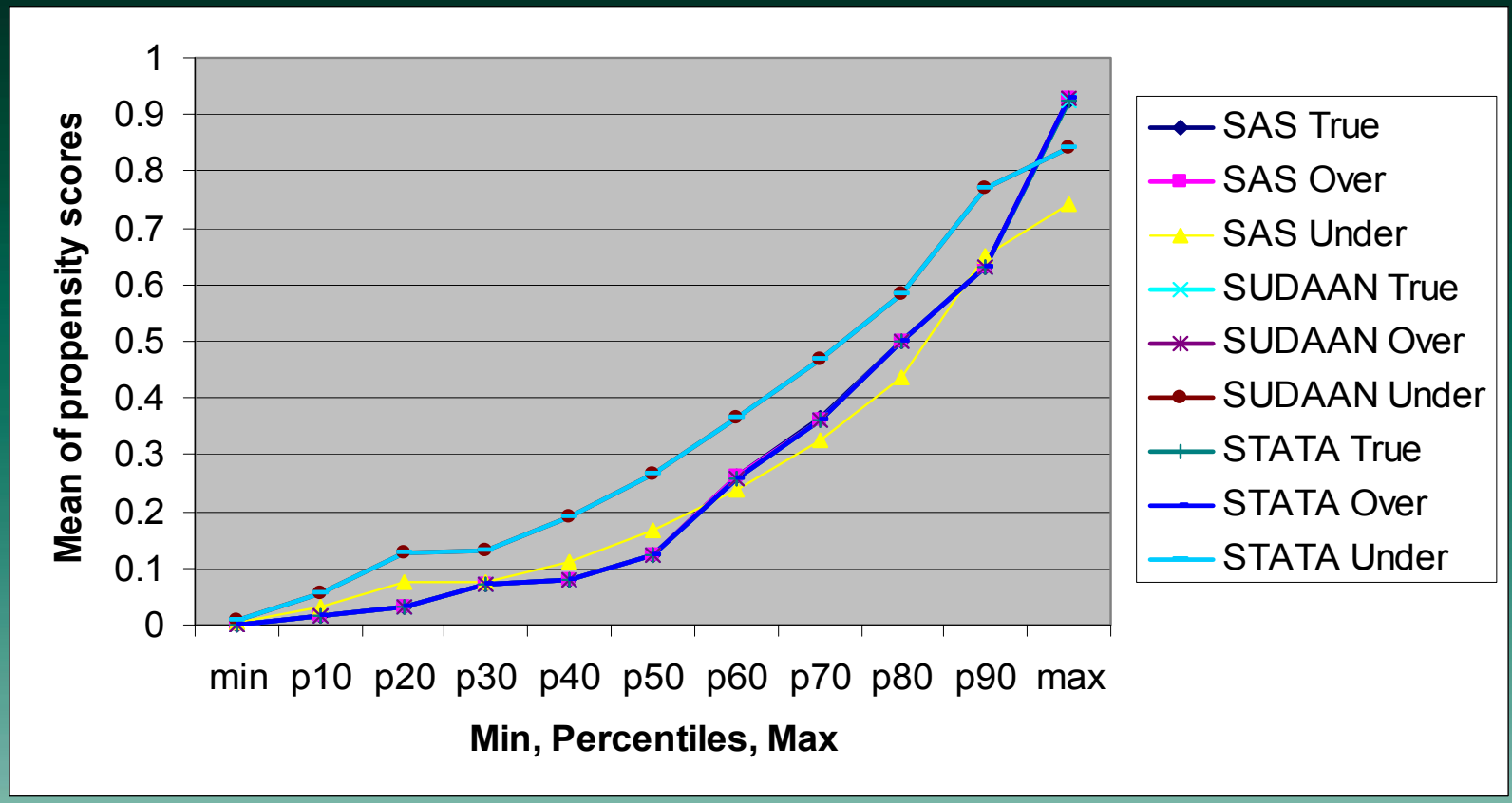
Simulation Study

Percentiles for *p*-values of Underspecified Model



Simulation Study

Means of Propensity Score Percentiles



Conclusion

- Simple random sample H-L and STATA GOF tests overstate the true *p*-value
- Chi-square rejection region works poorly for GOF test for complex survey data
- Underspecified model can be detected by low *p*-value in the GOF test
- Minor model overspecification does not affect distribution of propensity scores, nor the weighting adjustment factors